WESLEY COLLEGE

By daring & by doing

YEAR 12 MATHEMATICS SPECIALIST
SEMESTER TWO 2017
QUESTIONS OF REVIEW 5: Differentiation with Applications

Name: Master.

Thursday 29th June   Time: 35 minutes   Mark /30

CAS free, scientific calculator allowed.

1. [7 marks – 1, 3 and 3]

A curve is defined by the equation $y^2 = 3xy - \frac{5x^2}{4}$

a) Verify that $P(2,5)$ lies on the curve

$$3xy - \frac{5x^2}{4} = 30 - 5 = 25 = y^2$$

b) Develop an expression for the gradient function $\frac{dy}{dx}$

$$2y \cdot \frac{dy}{dx} = 3y + 3x\frac{dy}{dx} - \frac{5x}{2}$$

$$\therefore \frac{dy}{dx}(2y - 3x) = 3y - \frac{5x}{2}$$

$$\therefore \frac{dy}{dx} = \frac{3y - \frac{5x}{2}}{2y - 3x} = \frac{6y - 5x}{4y - 6x}$$

c) Determine an equation for the normal to the curve at $P(2,5)$.

$$\left.\frac{dy}{dx}\right|_{(2,5)} = \frac{30 - 10}{20 - 12} = \frac{5}{2}$$

$$\therefore \text{normal is } y = -\frac{2x}{5} + c \text{ at } (2,5)$$

$$c = 5 + \frac{4}{5} = \cancel{10}\ 5\tfrac{4}{5}$$

$$\therefore y = -\frac{2x}{5} + \frac{29}{5}$$

2. [4 marks]

A particle with displacement x has velocity $v = 3\sqrt{x}$.

Show that the acceleration is constant and evaluate this constant.

$$\frac{dx}{dt} = 3\sqrt{x}$$

$$\therefore a = \frac{d^2x}{dt^2} = 3 \cdot \frac{1}{2} x^{-\frac{1}{2}} \cdot \frac{dx}{dt}$$

$$= \frac{3}{2\sqrt{x}} \cdot 3\sqrt{x} = \frac{9}{2}$$ the req'd constant

3. [~~5~~ 6 marks – ~~2~~ 3, 2 and 1]

A second charged particle in a magnetic field has velocity $v = 8\sqrt{x}$ cm/sec when it has travelled x cm from rest.

a) Show that, if ∂x and ∂v represent small changes in x and v respectively, $\partial v \approx \frac{32\partial x}{v}$

$$\partial v = \frac{dv}{dx}\partial x$$

$$= \frac{4}{\sqrt{x}} \cdot \partial x$$

$$= \frac{32}{v}\partial x$$

b) Estimate the percentage change in x needed to reduce v by 4%.

$$\frac{\partial v}{v} = \frac{1}{2}\frac{\partial x}{x} \Rightarrow \frac{\partial x}{x} = 8\%$$ reduction

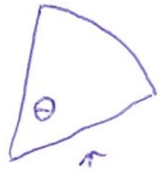
c) Explain whether this is a valid method to estimate the effect of a 50% change in x.

No, 50% is not small / incremental.

4. [~~9~~ 8 marks – ~~3~~ 4, ~~2~~ 1, ~~2~~ 1 and 2]

A damaged oil tanker is leaking oil into the sea. A current pushes the spreading oil into the shape of a sector of a circle, with radius r and sector angle θ. Both r and θ change with time.

The area of a sector is given by $A = \frac{1}{2}r^2\theta$.



a) Show that $\frac{d\theta}{dt} = \frac{2}{r}\left(\frac{1}{r}\frac{dA}{dt} - \theta\frac{dr}{dt}\right)$

$$\therefore \frac{dA}{dt} = r\theta\frac{dr}{dt} + \frac{1}{2}r^2\frac{d\theta}{dt}$$

$$\therefore \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{dA}{dt} - r\theta\frac{dr}{dt}$$

$$\therefore \frac{d\theta}{dt} = \frac{2}{r^2}\left(\frac{dA}{dt} - r\theta\frac{dr}{dt}\right) = \frac{2}{r}\left(\frac{1}{r}\frac{dA}{dt} - \theta\frac{dr}{dt}\right)$$

The radius of the oil spill is increasing at 2 m per minute and the area at 2π m^2 per minute.

When the oil spill has a radius of 6 m:

b) determine the area at this instant

$t = 3$

$A = 6\pi \ m^2$

c) find the exact value of θ at this instant

$$6\pi = \frac{1}{2}\cdot 6^2 \cdot \theta$$

$$\Rightarrow \theta = \frac{\pi}{3}$$

d) calculate the rate of change of θ at this instant.

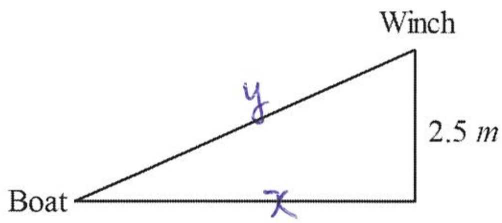
$$\frac{d\theta}{dt} = \frac{2}{6}\left(\frac{1}{6}\cdot 2\pi - \frac{\pi}{3}\cdot 2\right)$$

$$= \frac{\pi}{9} - \frac{2\pi}{9} = -\frac{\pi}{9}$$

i.e. decreasing at $\frac{\pi}{9}$ rad/min

5. [5 marks]

A small boat is being hauled towards a wharf by a winch mounted 2.5 m above water level. The winch is pulling the connecting rope at a rate of 0.06 m per second.



How fast is the boat moving horizontally when there is 6.5 of rope between the boat and the winch?

$$y^2 = x^2 + 2.5^2$$

$y = 6.5$

$\Rightarrow x = 6$

$(5, 12, 13\ \Delta)$

$$2y \frac{dy}{dt} = 2x \frac{dx}{dt}$$

$$\therefore \frac{dx}{dt} = \frac{y}{x} \cdot \frac{dy}{dt}$$

$$= \frac{6.5}{6} \times 0.06$$

$$= 0.065 \text{ m/sec.}$$